كلية التقنية الالكترونية - طرابلس
College of Electronic Technology - Tripoli

القسم: الاتصالات — أسئلة الامتحان النهائي — المادة: كهرومغناطيسية I

لطلبة الفصل: الخامس — رمز المادة: CM321 — التاريخ: 2022/11/12

للفصل الدراسي : ربيع2022 — اسم الأستاذ: أ. مبروكة العاقل — الزمن: ساعتان

اسم الطالب:.. رقم القيد .... المجموعة: ....

The total mark of this exam this: 60

This exam paper consists of 2 pages

Question 1: [10 M]

State whether the following sentences are true (✓) or false (✗):

1- Vector scaling refers to the change in magnitude of a vector but scaling cannot change the direction (other than flipping). (✓)

2- The force between two-point charges Q_1 and Q_2 is ~~directly~~ inversely proportional to the square of the distance R between them. (✗)

3- Electrostatic fields are produced by static charge distribution. (✓)

4- The electrostatic Field is time-invariant Field. (✓)

5- A vector field A is said to be irrotational if the ~~divergence~~ curl of the vector is zero. (✗)

6- Solenoidal fields have zero divergence. (✓)

7- An infinite sheet of charge in the xz-plane has electric field intensity E in the ~~x~~ y- direction. (✗)

8- The electric field intensity E of an infinite line charge is ~~parallel~~ perpendicular to the line charge. (✗)

9- The electric field intensity E at point p due to an infinite charged sheet is independent of the distance between the sheet and point p. (✓)

10- The electric flux density **D** is not a function of the medium permittivity ε. (✗)

$$D = \varepsilon E = \frac{\varepsilon Q}{4\pi\varepsilon R^2} = \boxed{\frac{Q}{4\pi R^2}}$$

Question 2: [5 M]

Chose the right answer:

1- Two vectors are identical (equal) if:
(a) The two vectors have the same direction.
(b) They have the same magnitude.
(c) They have the same magnitude and direction.

2- The unit vector:
(a) Has magnitude 1 and is a scalar.
(b) Has magnitude 1 and is a vector.
(c) As in (b) but also must be in the direction of a given vector.

3- If two vectors have identical unit vectors:
(a) The two vectors are identical.
(b) The two vectors are parallel but not necessarily of the same magnitudes.
(c) The two vectors are parallel but can point in opposite directions.

4- Which of the following statements are correct?
(a) The vector product of two perpendicular vectors is zero.
(b) The scalar product of two perpendicular vectors is zero.
(c) The scalar product of two parallel vectors is zero.

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5- Two vectors are perpendicular to each other if :

(a) Their vector product is zero.

(b) Their unit vectors are identical.

(c) Their scalar product is zero.

Question 3: [25 M]

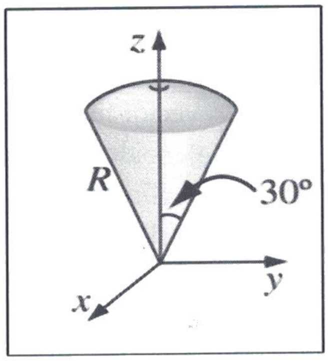
i. Given the vector function $\vec{E} = (y - c_1 z)\hat{a}_x + (c_2 x - 2z)\hat{a}_y + (c_3 y + z)\hat{a}_z$

Determine constants c_1, c_2, c_3 If $\vec{E}$ is conservative.

ii. Transform the vector $\vec{A} = 3\hat{a}_x + 4\hat{a}_y + 5\hat{a}_z$ at point (3,4,5) into spherical coordinates.

iii. Check the divergence theorem for the vector field

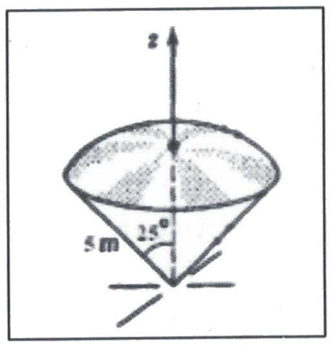
$\vec{F} = r^2 sin(\theta)\hat{a}_r + 4r^2 cos(\theta)\hat{a}_\varphi$,using the volume of the "ice-cream cone" shown in Figure (the top surface is spherical, with radius R and centered at the origin).



iv. A vector field is given by $\vec{G} = 15r\,\hat{a}_\varphi$.

Verify Stoke's theorem for a segment of a spherical surface defined by :

$$r = 5m, 0 \le \theta \le 25°, 0 \le \varphi \le 2\pi$$



Question 4: [20 M]

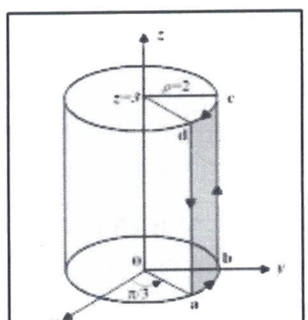
i. If $V = \frac{cos(\theta)}{r^2}\,V$, Find $\vec{E}$ and ρ_v.

ii. If $\vec{D} = \frac{5}{r^2}\,\hat{a}_r - r^3\varphi sin(\theta)\hat{a}_\varphi\ C/m^2$ for a sphere of radius a . What is Q in the sphere?

iii. $If\ \vec{H} = \frac{7.5*10^6}{\pi\rho}\cos(\varphi)\hat{a}_\rho\ A/m$ Find the magnetic flux crossing the surface of a cylinder defined by $\rho = 2, \frac{\pi}{3} \le \varphi \le \frac{\pi}{2}, 0 \le z \le 3$,

Then find the <u>**total flux for the closed surface of the whole cylinder**</u>.

Use $\Psi = \int \vec{B}.\vec{dS}$, $\mu_o = 4\pi * 10^{-7} H/m$



Q3 i:- $\vec{E} = (y - c_1 z)\hat{a}_x + (c_2 x - 2z)\hat{a}_y + (c_3 y + z)\hat{a}_z$

For conservative or irrotational $\Rightarrow \nabla \times \vec{E} = 0$

$$\nabla \times \vec{E} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y - c_1 z & c_2 x - 2z & c_3 y + z \end{vmatrix} = 0$$

$$(c_3 - 2)\hat{a}_x \quad - c_1 \hat{a}_y \quad + (c_2 - 1)\hat{a}_z = 0$$

$$c_3 - 2 = 0 \Rightarrow c_3 = 2 \quad , \quad c_1 = 0 \quad , \quad c_2 = 1$$

(ii) $\vec{A} = 3\hat{a}_x + 4\hat{a}_y + 5\hat{a}_z$ at $(3,4,5)$ To spherical

$P(3,4,5)$ $\qquad \sqrt{(3)^2 + (4)^2 + (5)^2} = 5\sqrt{2}$

$$\theta = \tan^{-1} \frac{\sqrt{(3)^2 + 4^2}}{5} = \tan^{-1}(1) = 45^\circ$$

$$\phi = \tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ$$

$$P(3,4,5) \Rightarrow (5\sqrt{2}, 45^\circ, 53.13^\circ)$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

$$A_r = 3\sin(\theta)\cos(\phi) + 4\sin(\theta)\sin(\phi) + 5\cos\theta$$
$$= 3(0.707)\left(\frac{3}{5}\right) + 4(0.707)\left(\frac{4}{5}\right) + 5(0.707)$$
$$= 1.8\left(\frac{1}{\sqrt{2}}\right) + 3.2\left(\frac{1}{\sqrt{2}}\right) + 5\left(\frac{1}{\sqrt{2}}\right) = \frac{10}{\sqrt{2}}$$

$$A_\theta = 3\cos(\theta)\cos(\phi) + 4\cos(\theta)\sin(\phi) - 5\sin(\theta)$$
$$= 3\left(\frac{1}{\sqrt{2}}\right)\left(\frac{3}{5}\right) + 4\left(\frac{1}{\sqrt{2}}\right)\left(\frac{4}{5}\right) - 5\left(\frac{1}{\sqrt{2}}\right) = 0$$

$$A_z = -3\sin(\phi) + 4\cos(\phi) = -3\left(\frac{4}{5}\right) + 4\left(\frac{3}{5}\right) = 0$$

$$A = \frac{10}{\sqrt{2}}\,\hat{a}_r = 5\sqrt{2}\,\hat{a}_r$$

(ii) $\vec{F} = r^2\sin(\theta)\,\hat{a}_r + 4r^2\cos(\theta)\,\hat{a}_\phi$

$$0 \le r \le R \;,\; 0^\circ \le \phi \le 2\pi \;,\; 0^\circ \le \theta \le 30^\circ$$

$$\nabla\cdot\vec{F} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^4\sin(\theta)\right) + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial\phi}\left(4r^2\cos(\theta)\right)$$

$$= 4r\sin(\theta)$$

$$\iiint_V \nabla\cdot\vec{F}\,dv = 4\int_0^{2\pi}\int_0^{30^\circ}\int_0^R r\sin(\theta)\,.\,r^2\sin(\theta)\,dr\,d\theta\,d\phi$$

$$= 4(2\pi)\int_0^R r^3\,dr\int_0^{30^\circ}\sin^2(\theta)\,d\theta$$

$$\int_0^{\frac{\pi}{6}}\sin^2\theta\,d\theta = \frac{1}{2}\int_0^{\frac{\pi}{6}}(1-\cos 2\theta)\,.\,d\theta = \frac{1}{2}\left[\theta - \frac{\sin(2\theta)}{2}\right]_0^{\frac{\pi}{6}} = \frac{\pi}{12} - \frac{\sqrt{3}}{8}$$

$$= 8\pi\left(\frac{R^4}{4}\right)(0.0453) = 0.0905\,R^4$$

$$= 2\pi R^4\left(\frac{\pi}{12} - \frac{\sqrt{3}}{8}\right)$$

$$= \frac{\pi R^4}{2}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)$$

Because we don't have component of $\hat{a}_\theta$ in vector $\vec{F}$ so we calculate flux in r direction

$$\oiint \vec{F}\,.\,ds = \iint_{S_1} F\,.\,ds_1 + \cancel{\iint_{S_2} F\,.\,ds_2}^{\text{zero}}$$

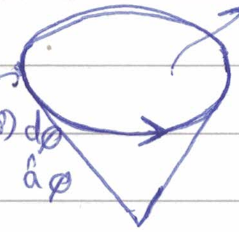
$$= \int_0^{2\pi}\int_0^{\frac{\pi}{6}} r^2\sin(\theta)\,(r^2\sin(\theta)\,d\theta\,d\phi$$

$$= r^4 (2\pi) \int_0^{\frac{\pi}{6}} \sin^2(\theta)\, d\theta = 2\pi R^4 \left(\frac{\pi}{12} - \frac{\sqrt{3}}{8}\right)$$

$$= \frac{\pi R^4}{2}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right)$$

$$= 0.0905 R^4 \ \#$$

(iv) $\vec{G} = 15 r\, \hat{a}_\phi$



$$\int_{\phi=0}^{2\pi} \vec{G}\cdot d\vec{l} = \int_0^{2\pi} 15 r^2 \sin(\theta)\, d\phi$$

$$= 15 (5)^2 \sin(25^\circ)(2\pi) = 995.77$$

$$\nabla \times \vec{G} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin(\theta)\,\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ 0 & 0 & 15 r^2 \sin(\theta) \end{vmatrix}$$

$$= \frac{1}{r^2 \sin(\theta)} \frac{\partial}{\partial \theta}\left(15 r^2 \sin(\theta)\right) \hat{a}_r - \frac{1}{r\sin(\theta)} \frac{\partial}{\partial r}\left(15 r^2 \sin(\theta)\right) \hat{a}_\theta$$

$$= 15 \frac{\cos(\theta)}{\sin(\theta)} \hat{a}_r - 30\, \hat{a}_\theta$$

$$\int (\nabla \times G)\cdot ds = \int_0^{2\pi}\int_0^{25^\circ} 15 r^2 \cos(\theta)\, d\theta\, d\phi$$

$$= (2\pi)\, 15 (5)^2 \int_0^{25^\circ} \cos(\theta)\, d\theta$$

$$= 2356.2 \left[\sin(\theta)\right]_0^{25^\circ} = 995.77 \ \#$$

Q4

(i) $V = \frac{\cos(\theta)}{r^2} \Rightarrow \vec{E} = -\nabla V$

$$= \frac{\partial}{\partial r}\left(-\frac{\cos(\theta)}{r^2}\right)\hat{a}_r - \frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{\cos(\theta)}{r^2}\right)\hat{a}_\theta$$

$$= \frac{\cos(\theta)}{r^3}\hat{a}_r + \frac{\sin(\theta)}{r^3}$$

$$= \frac{\cos\theta\,\hat{a}_r + \sin(\theta)\,\hat{a}_\theta}{r^3}$$

$$\rho_v = \nabla.\vec{D} = \varepsilon\nabla.\vec{E} \Rightarrow$$

$$\nabla.\vec{E} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{\cos(\theta)}{r}\right) + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial\theta}\left(\frac{\sin^2\theta}{r^3}\right)$$

$$= \frac{-\cos(\theta)}{r^4} + \frac{1}{r^4\cancel{\sin(\theta)}}(2\cancel{\sin(\theta)}\cos(\theta))$$

$$= \frac{\cos(\theta)}{r^4}$$

$$\rho_v = \frac{\varepsilon\cos(\theta)}{r^4} = \frac{8.85\times10^{-12}\cos(\theta)}{r^4}\ C/m^3$$

(ii) $\vec{D} = \frac{5}{r^2}\hat{a}_r - r^3\phi\sin(\theta)\,\hat{a}_\phi$

$$\rho_v = \nabla.\vec{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(5) + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial\phi}\left(r^3\phi\sin(\theta)\right)$$

$$= -r^2\ C/m^3$$

$$Q = \int_0^{2\pi}\int_0^{\pi}\int_0^{a}\rho_v\,.dv = \int_0^{2\pi}\int_0^{\pi}\int_0^{a} -r^4\sin(\theta)\,dr\,d\theta\,d\phi$$

$$= \frac{-r^5}{5}\Big|_0^a \; -\cos(\theta)\Big|_0^\pi \; (2\pi)$$

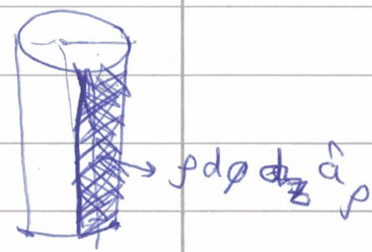
$$= \frac{-2\pi a^5}{5}\ C$$

(iii) $\vec{B} = \mu_0 \vec{H} = \dfrac{4\pi \times 10^{-7} \times 7.5 \times 10^{6}}{\pi \rho} \cos(\phi)\, \hat{a}_\rho$

$$= \frac{3}{\rho} \cos(\phi)\, \hat{a}_\rho \quad Wb/m^2 \ (T).$$

$$\Psi = \int_S B \cdot ds$$

$$= 3 \int_0^3 \int_{\pi/3}^{\pi/2} \cos(\phi)\, d\phi\, dz$$



$$= 3\, z\Big|_0^3 \ \sin(\phi)\Big|_{\pi/3}^{\pi/2} = 9\left(1 - \frac{\sqrt{3}}{2}\right) Wb$$

$$= 1.2057\ Wb$$

For total flux of the closed surface:

Maxwell's equation: $\oint B \cdot ds = \underline{\underline{0}}$

or $\Psi = \oint_S B \cdot ds = \underbrace{\cancel{\oint_{upper} B \cdot ds}}_{zero} + \underbrace{\cancel{\oint_{Lower} B \cdot ds}}_{zero} + \oint_{side} B \cdot ds$

$$= 3 \int_0^3 \int_0^{2\pi} \cos(\phi)\, d\phi\, dz$$

$$= 3\, z\Big|_0^3 \ \sin(\phi)\Big|_0^{2\pi} = \underline{\underline{0}} \quad \#$$